Nonleptonic charmless two-body B decays involving tensor mesons in the covariant light-front approach

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Abstract

We reanalyzed nonleptonic charmless two-body B decays involving tensor mesons in final state motivated by the disagreement between current experimental information and theoretical predictions obtained in ISGW2 model for some $\mathcal{B}(B \to P(V)T)$ (where P, V and T denote a pseudoscalar, a vector and a tensor meson, respectively). We have calculated branching ratios of charmless $B \to PT$ and $B \to VT$ modes, using $B \to T$ form factors obtained in the covariant light-front (CLF) approach and the full effective Hamiltonian. We have considered the $\eta - \eta'$ two-mixing angle formalism for $B \to \eta^{(\prime)}T$ channels, which increases branching ratios for these processes. Our predictions obtained in the CLF approach are, in general, greater than those computed in the framework of the ISGW2 model and more favorable with the available experimental data. Specifically, our results for exclusive channels $B \to \eta K_2^*(1430)$ and $B \to \phi K_2^*(1430)$ are in agreement with recent experimental information.

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I. INTRODUCTION

In this work we have re-examined the production of tensor mesons in nonleptonic charmless two-body B decays motivated by the discrepancy between experimental branching ratios for $B \to VT$ and $B \to PT$ modes, reported recently by BaBar Collaboration [1, 2, 3] and in Particle Data Group (PDG) [4], respectively, and theoretical predictions obtained in [5] using the ISGW2 model [6] for evaluating $B \to T$ form factors (see Table I). Keeping in mind that the ISGW2 model presents difficulties in the low-q^2 region, we have computed branching ratios for charmless $B \to PT$ and $B \to VT$ modes using the covariant light-front (CLF) approach [7], obtaining predictions more favorable with experimental data.

There is another interest to study $B \to VT$ decays is that they offer a good scenario in order to investigate about the fraction of longitudinal and transverse decays, similarly to $B \to V_1V_2$ decays [8, 9]. Nowadays, decays with tensor mesons in final state is an area where data is well ahead of the theory. For a recent review about charmless hadronic B-meson decays see Ref. [10].

At the theoretical level, there are some works based on quark models that had obtained branching ratios of nonleptonic two-body B decays including tensor mesons in final state. Initially, Refs. [11, 12] calculated at tree level branching ratios of $B \to PT$ channels using the nonrelativistic ISGW model [13]. Ref. [12] also computed branching ratios of $B \to VT$ modes in this model. After, Refs. [14, 15], obtained branchings of charmless $B \to P(V)T$ channels, using the same model, but considering all the contributions from the effective Hamiltonian. Ref. [5] is the most recent comprehensive and systematic study about exclusive charmless $B \to P(V)T$ decays. In this work, authors calculated branching ratios of these modes considering the full effective Hamiltonian and using the improved ISGW2 model [6] for evaluating $B \to T$ form factors. In ISGW2 model, branching ratios are enhanced by about an order of magnitude compared to previous estimates using ISGW model. Recently, $\mathcal{B}(B^0 \to \phi K_2^{*0})$ was obtained using the light-front quark model (LFQM) [8]. This prediction is more favorable with experimental data.

At the present, $B \to T$ form factors have been calculated in a few quark models: in the ISGW2 model [6] and in the CLF approach [7]. Numerical values for form factors in both models are different. However, the ISGW2 model is not expected to be reliable in the low-q^2 region, in particular, at the maximum $q^2 = 0$ recoil point where the final-state meson could be highly relativistic. In general, theoretical predictions obtained in Ref. [5] using the ISGW2 model disagree with recent experimental data. In Table I, we display available experimental branching ratios (see third column) for some exclusive charmless $B \to P(V)T$ decays and the respective theoretical predictions reported in Ref. [5] using the ISGW2 model (here $\xi = 1/N_c$, where N_c is the color number). We can see that, in general, theoretical predictions are lower that experimental data.

Our aim in this work is to perform a comprehensive and systematic study about $B \to P(V)T$ decays but taking $B \to T$ form factors from CLF approach [7]. Additionally, we have included the $\eta - \eta'$ two-mixing angle formalism for $B \to \eta^{(\prime)} a_2(K_2^*)$ modes, which increases considerably branching ratios. This mixing has not been considered in previous works [5, 14].

Table I. Comparison between available experimental data for branching ratios (in units of 10^{-6}) of charmless $B \to P(V)T$ decays and theoretical predictions of Ref. [5] using ISGW2 model.

	Modes	$\mathcal{B}_{exp}$	Ref. [5]		
			$\xi = 0.1$	$\xi = 0.3$	$\xi = 0.5$
$B \to PT$	$B^+ \to \eta K_2^*(1430)^+$	(9.1 ± 3.0) [4]	0.256	0.031	0.028
	$B^+ \to \pi^+ f_2(1270)$	(8.2 ± 2.5) [4]	3.284	2.874	2.491
	$B^+ \to K^+ f_2(1270)$	$(9.1^{+0.4}_{-0.5})$ [4]	0.394	0.344	0.298
	$B^0 \to \eta K_2^*(1430)^0$	(9.6 ± 2.1) [4]	0.237	0.029	0.026
$B \to VT$	$B^0 \to \phi K_2^*(1430)^0$	$(7.8 \pm 1.1 \pm 0.6)$ [1]	0.517	2.024	4.532
	$B^\pm \to \phi K_2^*(1430)^\pm$	$(8.4 \pm 1.8 \pm 0.9)$ [2]	0.557	2.18	4.881
	$B^+ \to \omega K_2^*(1430)^+$	$(21.5 \pm 3.6 \pm 2.4)$ [3]	2.392	0.112	0.789
	$B^0 \to \omega K_2^*(1430)^0$	$(10.1 \pm 2.0 \pm 1.1)$[3]	2.221	0.104	0.732

This paper is organized as follows: in Sec. II, we discuss about the effective weak Hamiltonian and factorization approach. Sec. III is dedicated to $B \to T$ form factors in the CLF approach. In Sec. IV, we present our numerical

results and conclusions are given in Sec. V. In appendix, we show explicitly expressions for decay amplitudes of $B \to \eta^{(\prime)} a_2$ and $B \to \eta^{(\prime)} K_2^*$ modes, incorporating the $\eta - \eta'$ two-mixing angle formalism.

II. WEAK EFFECTIVE HAMILTONIAN AND FACTORIZATION SCHEME

The effective $\Delta B = 1$ weak Hamiltonian H_{eff} for two-body charmless hadronic B decays is [16]:

$$\begin{aligned} H_{eff} &= \frac{G_F}{\sqrt{2}}\Bigg[V_{ub}V_{uq}^*\Big(C_1(\mu)O_1^u(\mu) + C_2(\mu)O_2^u(\mu)\Big) + V_{cb}V_{cq}^*\Big(C_1(\mu)O_1^c(\mu) + C_2(\mu)O_2^c(\mu)\Big) \\ &\quad -V_{tb}V_{tq}^*\left(\sum_{i=3}^{10} C_i(\mu)O_i(\mu)\right)\Bigg] + h.c. \,, \end{aligned} \tag{1}$$

where $q = d, s$, G_F is the Fermi constant, $C_i(\mu)$ are the Wilson coefficients evaluated at the renormalization scale μ, and V_{ij} is the respective Cabibbo-Kobayashi-Maskawa (CKM) matrix element. We show below the local operators O_i for $b \to d, s$ transitions:

- current-current (tree) operators

$$\begin{aligned} O_1^u &= (\bar{q}_\alpha u_\alpha)_{V-A} \cdot (\bar{u}_\beta b_\beta)_{V-A} \\ O_2^u &= (\bar{q}_\alpha u_\beta)_{V-A} \cdot (\bar{u}_\beta b_\alpha)_{V-A} \\ O_1^c &= (\bar{q}_\alpha c_\alpha)_{V-A} \cdot (\bar{c}_\beta b_\beta)_{V-A} \\ O_2^c &= (\bar{q}_\alpha c_\beta)_{V-A} \cdot (\bar{c}_\beta b_\alpha)_{V-A} \end{aligned} \tag{2}$$

- QCD penguin operators

$$\begin{aligned} O_{3(5)} &= (\bar{q}_\alpha b_\alpha)_{V-A} \cdot \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A(V+A)} \\ O_{4(6)} &= (\bar{q}_\alpha b_\beta)_{V-A} \cdot \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A(V+A)} \end{aligned} \tag{3}$$

- electroweak penguin operators

$$\begin{aligned} O_{7(9)} &= \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \cdot \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A(V-A)} \\ O_{8(10)} &= \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A} \cdot \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A(V-A)}, \end{aligned} \tag{4}$$

where $(\bar{q}_1 q_2)_{V\mp A} \equiv \bar{q}_1\gamma_\mu(1 \mp \gamma_5)q_2$, and α and β are $SU(3)$ color indices. The sums run over the active quarks at the scale $\mu = \mathcal{O}(m_b)$, i.e. $q' = u, d, s, c$.

In order to obtain branching ratios of two-body nonleptonic $B \to M_1 M_2$ decays it is necessary to evaluate the hadronic matrix element involving four-quark operators $\langle M_1 M_2|\mathcal{H}_{eff}|B\rangle$. In the framework of factorization approach, it can be expressed as the product of two matrix elements of single currents, which are governed by decay constants and form factors. The hadronic matrix element is renormalization scheme and scale independent [17] while the Wilson Coefficients are renormalization scheme and scale dependent.

For solving the aforementioned scale problem, Refs. [18, 19] proposed to extract the μ dependence from the matrix element $\langle O_i(\mu)\rangle$ and combine it with the μ-dependent Wilson coefficients $C_i(\mu)$ to form μ-independent effective Wilson coefficients c_i^{eff}. We have taken numerical values for them reported in Table I of Ref. [20]. They were

calculated using the naive dimensional regularization scheme.

It is known that the effective Wilson coefficients c_i^{eff} appear in the factorizable decay amplitudes as linear combinations. It allows to define the effective coefficients a_i, which are renormalization scale and scheme independent, expressed by

$$
\begin{aligned}
a_i &\equiv c_i^{eff} + \frac{1}{N_c} c_{i+1}^{eff} \ (i = odd), \\
a_i &\equiv c_i^{eff} + \frac{1}{N_c} c_{i-1}^{eff} \ (i = even) \ ,
\end{aligned} \tag{5}
$$

where the index i runs over $(1, ..., 10)$ and $N_c = 3$ is the number of colors. Phenomenologically, nonfactorizable contributions to the hadronic matrix element are modeled by treating N_c as a free parameter and its value can be extracted from experiment. In this work we have used numerical values for a_i coefficients reported in Table II of Ref. [20].

III. $B \to T$ FORM FACTORS IN THE CLF APPROACH

In order to obtain numerical values of branching ratios of $B \to P(V)T$ decays in the framework of generalized factorization, we need to compute the hadronic matrix element $\langle T|J^\mu|B\rangle$. We have used the parametrizations given in Ref. [13]:

$$
\begin{aligned}
\langle T|V^\mu|B\rangle &= ih(q^2)\varepsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\alpha}p_B^\alpha(p_B+p_T)_\rho(p_B-p_T)_\sigma, \\
\langle T|A^\mu|B\rangle &= k(q^2)\epsilon^{*\mu\nu}(p_B)_\nu + \epsilon^*_{\alpha\beta}p_B^\alpha p_B^\beta \\
&\quad \left[b_+(q^2)(p_B+p_T)^\mu + b_-(q^2)(p_B-p_T)^\mu\right] ,
\end{aligned} \tag{6}
$$

where V^μ and A^μ denote a vector and an axial-vector current, respectively, $\epsilon_{\nu\alpha}$ is the polarization tensor of tensor meson, p_B and p_T are the momentum of the B meson and the tensor meson, respectively, and $h, k, b_\pm$ are form factors for the $B \to T$ transition; h is dimensionless and $k, b_\pm$ have dimension of GeV^{-2}.

At the moment, only two models[1] provide a systematical estimate of $B \to T$ form factors: the ISGW model [6, 13] and CLF quark model [22]. Branching ratios for $B \to P(V)T$ modes using the ISGW2 model were calculated in Ref. [5]. In general, these predictions present some discrepancies with experimental data (as illustration, see Table I). Thus, in this work we have used numerical values for form factors $h, k, b_\pm$, obtained in CLF quark model [7]. This work has extended the covariant analysis of the light-front approach [22] to even-parity, p-wave mesons.

A LFQM can give a relativistic treatment of the movement of the hadron and also provides a fully description of the hadron spin. The light-front wave functions are independent of the hadron momentum and therefore explicitly Lorentz invariant. In the CLF quark model, the spurious contribution, which depends on the orientation of the light-front, is cancelled by the inclusion of the zero mode contribution, and becomes irrelevant in the decay constants and the form factor, so that the result is guaranteed to be covariant and more self consistent. Recently, this model has been used in several works: Ref. [23] investigated about semileptonic decays of B_c meson including s-wave and p-wave mesons in final state; Ref. [24] studied nonleptonic $B_c^- \to X(3872)\pi^-(K^-)$ modes; Ref. [25] worked with two-photon annihilation $P \to \gamma\gamma$ and magnetic dipole transition $V \to P\gamma$ processes for the ground-state heavy quarkonium within the CLF approach; Ref. [26] investigated about radiative $B \to (K^*, K_1, K_2^*)\gamma$ channels in the same framework; and Ref. [8] examined $B \to (K_0^*(1430), K_2^*(1430))\phi$ in the LFQM. In general, predictions in these works are more favorable with available experimental data.

In CLF approach form factors are explicit functions of q^2 in the space-like region and then analytically extend them to the time-like region in order to determine physical form factors at $q^2 \geq 0$. They are parametrized and reproduced in the three-parameter form [7]:

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[1] Recently Ref. [21] calculated $B \to K_2^*$ form factors using large energy effective theory (LEET) techniques.

$$F(q^2) = \frac{F(0)}{1 - aX + bX^2}, \tag{7}$$

whit $X = q^2/m_B^2$. Parameters a, b and $F(0)$ (form factor at the zero momentum transfer) for $B \to a_2(1320)$ and $B \to K_2^*(1430)$ transitions, which are $B \to T$ transitions required in this work, are displayed in Tables VI and VII of Ref. [7]. In Table II, we have summarized these numerical values.

Table II. Form factors for $B \to a_2(1320)$ and $B \to K_2^*(1430)$ transitions obtained in the CLF model [7] are fitted to the 3-parameter form in Eq.(7).

	$B \to a_2$			$B \to K_2^*$		
F	$F(0)$	a	b	$F(0)$	a	b
h	0.008	2.20	2.30	0.008	2.17	2.22
k	0.031	-2.47	2.47	0.015	-3.70	1.78
b_+	-0.005	1.95	1.80	-0.006	1.96	1.79
b_-	0.0016	-0.23	1.18	0.002	0.38	0.92

Model predictions for $B \to T$ form factors in the CLF quark model [7] are different from those in the improved version of ISGW model [6]. Form factors at small q^2 obtained in the CLF and ISGW2 models agree within 40% [7]. However, when q^2 increases $h(q^2)$, $|b_+(q^2)|$ and $b_-(q^2)$ increase more rapidly in the light-front model than those in the ISGW2 model [7]. Another important fact is that the behavior of the form factor k in both models is different (see Table II of Ref. [8]): specifically, for $B \to K_2^*\phi$, $k(m_\phi^2)$ is bigger in ISGW2 model than in LFQM: $[k(m_\phi^2)|_{\rm ISGW2}]/[k(m_\phi^2)|_{\rm LFQM}] = 16.69$.

On the other hand, we also need to evaluate the matrix element of the current between the vacuum and final pseudoscalar (P) or vector (V) mesons. It can be expressed in terms of the respective decay constants $f_{P(V)}$, in the form

$$\begin{aligned} \langle P(p_P)|A_\mu|0\rangle &= i f_P q_\mu \\ \langle V(p_V,\epsilon)|V_\mu|0\rangle &= f_V m_V \epsilon_\mu, \end{aligned} \tag{8}$$

where $q_\mu = (p_B - p_T)_\mu$ and ϵ_μ is the vector polarization of V meson. Finally, it is important to note that the polarization tensor $\epsilon_{\mu\nu}$ of a $^3P_2$ tensor meson satisfies the relations

$$\epsilon_{\mu\nu} = \epsilon_{\nu\mu}, \qquad \epsilon^\mu_\mu = 0, \qquad p_\mu \epsilon^{\mu\nu} = p_\nu \epsilon^{\mu\nu} = 0. \tag{9}$$

Therefore,

$$\langle 0|(V-A)_\mu|T\rangle = a\epsilon_{\mu\nu}p^\nu + b\epsilon^\nu_\nu p_\mu = 0, \tag{10}$$

and hence the decay constant of the tensor meson vanishes, i.e., the tensor meson can not be produced from the vacuum. Thus, decay amplitudes for $B \to PT$, VT processes can be considerably simplified compared to those for two-body charmless B decays such as $B \to PP$, PV, and VV [18, 19], and $B \to PA$, AV, and AA [20].

IV. NUMERICAL RESULTS

In this section we present numerical inputs that are necessary to obtain our predictions, and numerical values for branching ratios of charmless $B \to PT$ and $B \to VT$ decays, using $B \to T$ form factors obtained in the CLF approach [7]. We used the following values of decay constants (in GeV units): $f_\pi = 0.1307$ and $f_K = 0.160$ for pseudoscalar mesons and $f_\rho = 0.216$, $f_\omega = 0.195$, $f_\phi = 0.236$ and $f_{K^*} = 0.221$ for vector mesons [4]. For decay constants of η and

η' mesons we adopt the $\eta - \eta'$ two-mixing angle formalism presented in [27, 28], which defines physical states η and η' in function of flavor octet and singlet, η_8 and η_0, respectively:

$$\begin{aligned} |\eta\rangle &= \cos\theta|\eta_8\rangle - \sin\theta|\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta|\eta_8\rangle + \cos\theta|\eta_0\rangle. \end{aligned} \tag{11}$$

Decay constants for η_8 and η_0 are given by $\langle 0|A^8_\mu|\eta_8\rangle = if_8 p_\mu$ and $\langle 0|A^0_\mu|\eta_0\rangle = if_0 p_\mu$. Assuming that η_8 and η_0 are

$$\begin{aligned} |\eta_8\rangle &= \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle, \\ |\eta_0\rangle &= \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle, \end{aligned} \tag{12}$$

they induce a two-mixing angle in the decay constants $f^q_{\eta^{(\prime)}}$, defined by $\langle 0|\bar{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}(p)\rangle = if^q_{\eta^{(\prime)}}p_\mu$:

$$\begin{aligned} f^u_{\eta'} &= \frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0, \\ f^s_{\eta'} &= -2\frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0, \end{aligned} \tag{13}$$

for the η' meson and

$$\begin{aligned} f^u_{\eta} &= \frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0, \\ f^s_{\eta} &= -2\frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0, \end{aligned} \tag{14}$$

for the η meson. From a complete phenomenological fit of the $\eta - \eta'$ mixing parameters in Ref. [28], we take $\theta_8 = -21.1^\circ$, $\theta_0 = -9.2^\circ$, $\theta = -15.4^\circ$, $f_8 = 165$ MeV and $f_0 = 153$ MeV. Using these numerical values in Eqs. (13) and (14), decay constants are $f^u_{\eta'} = 61.8$ MeV, $f^s_{\eta'} = 138$ MeV, $f^u_{\eta} = 76.2$ MeV and $f^s_{\eta} = -110.5$ MeV. For including the η_c in the mixing framework, we use decay constants defined by $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta^{(\prime)}\rangle = if^c_{\eta^{(\prime)}}p_\mu$. Ref. [28] obtained $f^c_\eta = -(2.4 \pm 0.2)$ MeV and $f^c_{\eta'} = -(6.3 \pm 0.6)$ MeV.

Masses and average lifetimes of neutral and charged B mesons were taken from [4]. The running quark masses are given at the scale $\mu \approx m_b$, since the energy released in B decays is of order m_b. We use $m_u(m_b) = 3.2$ MeV, $m_d(m_b) = 6.4$ MeV, $m_s(m_b) = 127$ MeV, $m_c(m_b) = 0.95$ GeV and $m_b(m_b) = 4.34$ GeV (see Ref. [29]).

We use Wolfenstein parameters λ, A, $\bar{\rho}$ and $\bar{\eta}$ [30] for parametrizing the CKM matrix:

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{15}$$

where $\rho = \bar{\rho}(1-\lambda^2/2)^{-1}$ and $\eta = \bar{\eta}(1-\lambda^2/2)^{-1}$. We take central values from the global fit for Wolfenstein parameters: $\lambda = 0.2257$, $A = 0.814$, $\bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$ [4].

For obtaining branching ratios, we have taken expressions for amplitudes of exclusive charmless $B \to PT$ ($P \neq \eta^{(\prime)}$) and $B \to VT$ decays given in appendices of Refs. [14] and [15], respectively. These expressions include all the contributions of H_{eff}. We do not consider decay amplitudes for $B \to \eta^{(\prime)}a_2$ and $B \to \eta^{(\prime)}K_2^*$ modes reported in Ref. [14]. We have worked with amplitudes displayed in the appendix, which include the $\eta - \eta'$ two-mixing angle formalism. This mixing increases considerably the respective branching ratios.

Our numerical results for branching ratios of exclusive charmless two-body $B \to PT$ and $B \to VT$ decays, in the CLF approach, are listed in Tables III - IV and V, respectively. Our predictions are compared with the work of Kim, Lee and Oh (KLO) [5], which evaluated form factors using the ISGW2 model. We have taken into account theoretical predictions of [5] with $m_s = 100$ MeV, $\xi = 1/N_c = 0.3$ and $\gamma = 65^\circ$ (see third column in Tables III and V, and fourth column in Table IV). In Table IV, we present our numerical predictions for $B \to \eta(\eta')T$ decays: results in second

column are obtained in the CLF approach using the amplitudes of Ref. [14] whereas results in third column are obtained in the same approach but using amplitudes showed in the appendix (these expressions include the $\eta{-}\eta'$ mixing).

We have not considered $B \to P(V)f_2$ and $B \to P(V)f_2'$ modes because Ref. [7] does not make predictions for $B \to f_2$ and $B \to f_2'$ transitions. They do not consider f_0, f_1 and f_2 mesons because their quark contents lying in the mass region of $1.3 - 1.7$ GeV [7].

We have analyzed the dependence of branching ratios for $B \to P(V)T$ about form factors. From expressions for decay widths given in Ref. [12] we can observe that $\mathcal{B}(B \to PT)$ and $\mathcal{B}(B \to VT)$ are quadratic functions of form factors k, $b_\pm$, and h, k, b_+, respectively. We have explored how is the behavior of these expressions when one changes smoothly numerical values of form factors. We have found that the strongest dependence of $\mathcal{B}(B \to P(V)T)$ is with respect to the form factor b_+. It is because of kinematical coefficients of b_+ are dominant in these expressions. Thus, the precise value of b_+ is an important test for both models (CLF and ISGW2).

Table III. Branching ratios (in units of 10^{-6}) for charmless $B \to PT$ decays, using form factors obtained in CLF model [7]. Our predictions are compared with the work of KLO [5].

Process	This work	KLO [5]
$B^+ \to \pi^+ a_2^0$	4.38	2.6
$B^+ \to \pi^0 a_2^+$	0.015	0.001
$B^+ \to K^+ a_2^0$	0.39	0.311
$B^+ \to \pi^0 K_2^{*+}$	0.15	0.09
$B^+ \to \bar{K}^0 K_2^{*+}$	7.84×10^{-4}	4×10^{-5}
$B^+ \to K^0 a_2^+$	0.015	0.011
$B^0 \to \pi^+ a_2^-$	8.19	4.88
$B^0 \to \pi^0 a_2^0$	0.007	0.0003
$B^0 \to K^+ a_2^-$	0.73	0.584
$B^0 \to \pi^0 K_2^{*0}$	0.13	0.084
$B^0 \to \bar{K}^0 K_2^{*0}$	7.15×10^{-4}	3×10^{-5}
$B^0 \to K^0 a_2^0$	0.014	0.005

Table IV. Branching ratios (in units of 10^{-6}) for charmless $B \to \eta^{(\prime)} a_2$ and $B \to \eta^{(\prime)} K_2^*$ decays without and with $\eta - \eta'$ mixing, using form factors from CLF model [7].

Process	Without $\eta - \eta'$ mixing	With $\eta - \eta'$ mixing	KLO [5]	Experiment [4]
$B^+ \to \eta a_2^+$	3.78	45.8	0.294	-
$B^+ \to \eta' a_2^+$	3.72	71.3	1.31	-
$B^+ \to \eta K_2^{*+}$	0.65	1.19	0.031	9.1 ± 3.0
$B^+ \to \eta' K_2^{*+}$	2.09	2.70	1.4	-
$B^0 \to \eta a_2^0$	1.77	25.2	0.138	-
$B^0 \to \eta' a_2^0$	7.20	43.3	0.615	-
$B^0 \to \eta K_2^{*0}$	0.59	1.09	0.029	9.6 ± 2.1
$B^0 \to \eta' K_2^{*0}$	1.91	2.46	1.3	-

Table V. Branching ratios for charmless $B \to VT$ decays, using form factors obtained in CLF model [7]. Our predictions are compared with the work of KLO [5].

Process	This work (10^{-6})	KLO [5] (10^{-6})	Experiment
$B^+ \to \rho^+ a_2^0$	19.34	7.342	
$B^+ \to \rho^0 a_2^+$	0.071	0.007	$< 7.2 \times 10^{-4}$ [4]
$B^+ \to \omega a_2^+$	0.14	0.01	
$B^+ \to \phi a_2^+$	0.019	0.004	
$B^+ \to K^{*+} a_2^0$	2.80	1.852	
$B^+ \to \rho^0 K_2^{*+}$	0.74	0.253	$< 1.5 \times 10^{-3}$ [4]
$B^+ \to \omega K_2^{*+}$	0.06	0.112	$(21.5 \pm 3.6 \pm 2.4) \times 10^{-6}$ [3]
$B^+ \to \phi K_2^{*+}$	9.24	2.18	$(8.4 \pm 1.8 \pm 0.9) \times 10^{-6}$ [2]
$B^+ \to \bar{K}^{*0} K_2^{*+}$	0.59	0.014	
$B^+ \to K^{*0} a_2^+$	8.62	4.495	
$B^0 \to \rho^+ a_2^-$	36.18	14.686	
$B^0 \to \rho^0 a_2^0$	0.03	0.003	
$B^0 \to \omega a_2^0$	0.07	0.005	
$B^0 \to \phi a_2^0$	0.009	0.002	
$B^0 \to K^{*+} a_2^-$	7.25	3.477	
$B^0 \to \rho^0 K_2^{*0}$	0.68	0.235	$< 1.1 \times 10^{-3}$ [4]
$B^0 \to \omega K_2^{*0}$	0.053	0.104	$(10.1 \pm 2.0 \pm 1.1) \times 10^{-6}$ [3]
$B^0 \to \phi K_2^{*0}$	8.51	2.024	$(7.8 \pm 1.1 \pm 0.6) \times 10^{-6}$ [1]
$B^0 \to \bar{K}^{*0} K_2^{*0}$	0.55	0.026	
$B^0 \to K^{*0} a_2^0$	4.03	2.10	

V. CONCLUSIONS

In this work we have re-analyzed charmless two-body hadronic $B \to PT$ and $B \to VT$ decays using form factors for $B \to T$ transitions from CLF approach, within the framework of generalized factorization. For $B \to \eta^{(\prime)} T$ decays we have considered the $\eta - \eta'$ two-mixing angle formalism which increases the respective branching ratios. Our results are compared with ones obtained using ISGW2 model [5] and with available experimental data. Predictions for exclusive $B \to \eta K_2^*(1430)$ and $B \to \phi K_2^*(1430)$ in CLF approach are in agreement with experiment data whereas results obtained in ISGW2 model are lower than them. Our main conclusions are:

- In general, our branching ratios predictions using the CLF approach in order to obtain form factors of $B \to T$ transitions are greater than previous work of KLO [5] using ISGW2 model. Predictions in CLF approach seems to be more favorable with the available experimental data. Some of this modes with branching ratios $\sim (10^{-5} - 10^{-6})$ could be measured at present asymmetric B factories, BABAR and Belle, as well as at future hadronic B experiments such as BTeV and LHC-b.

- Our numerical results for penguin processes $B^0 \to \phi K_2^*(1430)^0$ and $B^+ \to \phi K_2^*(1430)^+$ (see Table V) are in agreement with BABAR results [1, 2] and with the prediction $\mathcal{B}(B^0 \to \phi K_2^{*0}) = 7.0 \times 10^{-6}$ reported recently in the theoretical work [8]. Predictions obtained in Ref. [5] are lower than these experimental data.

- The inclusion of $\eta - \eta'$ mixing effects in amplitudes of $B \to \eta^{(\prime)} a_2$ and $B \to \eta^{(\prime)} K_2^*$ modes, increases considerably their branching ratios (see third column in Table IV), so $B \to \eta^{(\prime)} T$ decays become important. Although our predictions for branching ratios of $B^{+,0} \to K_2^{*+,0} \eta$ are lower than experimental data they are more favorable than those of Ref. [5], which are too lower. Let us mention that even using amplitudes of Ref. [14] for these modes but working in the CLF framework, the predictions (see second column in Table IV) are bigger than those obtained in ISGW2 model [5].

- In charmless $B \to \pi(K)T$ modes, the bigger discrepancy between predictions of both models (CLF and ISGW2) appears in the exclusive channels $B^{+,0} \to \pi^0 a_2^{+,0}$ and $B^{+,0} \to \bar{K}^0 K_2^{*+,0}$. The ratio $[\mathcal{B}(B \to \pi^0(\overline{K^0}) a_2(K_2^*))_{\rm CLF}]/[\mathcal{B}(B \to \pi^0(\overline{K^0}) a_2(K_2^*))_{\rm ISGW2}]$ is $\sim$ (15 - 23).

- In charmless $B \to VT$ modes (see Table V), the bigger discrepancy between predictions of both models arises from exclusive penguin channels $B^{+,0} \to \bar{K}^{*0} K_2^{+,0}$. In this case, the branching in the CLF approach is $\sim$ (21 - 42) times the one in ISGW2 model. On the other hand, branchings of exclusive $B^{+,0} \to \rho^0(\omega) a_2^{+,0}$ modes, in CLF model, are $\sim$ (10 - 14) times than the ones in ISGW2 model. Branchings of $B \to \omega K_2^*$ in CLF approach are the only lower predictions than those obtained in ISGW2 model. However, predictions in both models are lower that experimental data.

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APPENDIX

In this appendix, we present expressions for decay amplitudes of $B \to \eta^{(\prime)} a_2$ and $B \to \eta^{(\prime)} K_2^*$ modes, including the $\eta - \eta'$ mixing formalism. Amplitudes displayed below must be multiplied by $i\, G_F\, \epsilon^*_{\mu\nu} p_B^\mu p_B^\nu / \sqrt{2}$. They are different of expressions displayed in Ref. [14].

$$
\begin{aligned}
\mathcal{A}(B^0 \to \eta^{(\prime)} a_2^0) =& f^u_{\eta^{(\prime)}} F^{B\to a_2}(m^2_{\eta^{(\prime)}}) \Bigg\{ V_{ub}V^*_{ud} a_2 + V_{cb}V^*_{cd} a_2 \left(\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} \right) \\
& - V_{tb}V^*_{td} \Bigg[a_4 + 2(a_3 - a_5) + \frac{1}{2}(a_7 - a_9 - a_{10}) - 2\left(a_6 - \frac{1}{2}a_8\right) R \left(1 - \frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}}\right) \\
& + (a_3 - a_5 + a_7 - a_9)\left(\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} \right) + (a_3 - a_5 - \frac{1}{2}(a_7 - a_9))\left(\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} \right) \Bigg] \Bigg\},
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\mathcal{A}(B^0 \to \eta^{(\prime)} K_2^{*0}) =& f^u_{\eta^{(\prime)}} F^{B\to K_2^*}(m^2_{\eta^{(\prime)}}) \Bigg\{ V_{ub}V^*_{us} a_2 + V_{cb}V^*_{cs} a_2 \left(\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} \right) \\
& - V_{tb}V^*_{ts} \Bigg[2(a_3 - a_5) + \frac{1}{2}(a_7 - a_9) + (a_3 - a_5 + a_7 - a_9)\left(\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} \right) \\
& + \Bigg[a_3 + a_4 - a_5 - \frac{1}{2}(a_7 - a_9 + a_{10}) - 2\left(a_6 - \frac{1}{2}a_8\right) X \left(1 - \frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}}\right) \Bigg] \Bigg] \left(\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}} \right) \Bigg\},
\end{aligned}
\tag{17}
$$

with

$$
R = \frac{m^2_{\eta^{(\prime)}}}{2m_s(m_b + m_d)}, \qquad X = \frac{m^2_{\eta^{(\prime)}}}{2m_s(m_b + m_s)},
\tag{18}
$$

and

$$
F^{B\to T}(m^2_{\eta^{(\prime)}}) \equiv k(m^2_{\eta^{(\prime)}}) + (m_B^2 - m_T^2) b_+(m^2_{\eta^{(\prime)}}) + m^2_{\eta^{(\prime)}} b_-(m^2_{\eta^{(\prime)}}),
\tag{19}
$$

where T stands for a_2 and K_2^*.

The factorized decay amplitudes of $B \to \eta^{(\prime)} a_2(K_2^*)$ are obtained from amplitudes of $B \to \eta^{(\prime)} \pi(K)$ (see for example appendix A of Ref. [18]) changing $(a_6 - a_8/2)$, $(a_7 - a_9)$ and $1/(m_b - m_q)$ by $-(a_6 - a_8/2)$, $-(a_7 - a_9)$ and $1/(m_b + m_q)$,

respectively, and keeping in mind that a tensor meson T can not be produced from vacuum in generalized factorization. It implies that $\mathcal{A}(B^+ \to \eta^{(\prime)} a_2^+) = \sqrt{2}\mathcal{A}(B^0 \to \eta^{(\prime)} a_2^0)$ and $\mathcal{A}(B^+ \to \eta^{(\prime)} K_2^{*+}) = \mathcal{A}(B^0 \to \eta^{(\prime)} K_2^{*0})$.

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